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Acceptance Charts for Low Data Environments

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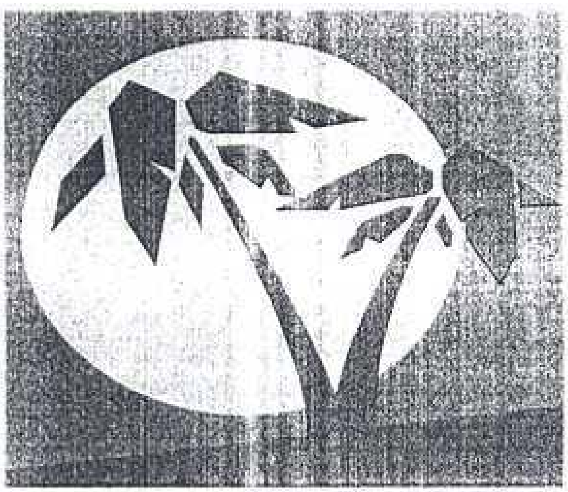

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PREFACE

Welcome to the thirty-first annual conference of the Northeast Decision Sciences Institute. The 2002 meeting is being held at the Caribe Hilton Hotel and Casino in San Juan, Puerto Rico. The Program includes 42 sessions of competitively judged papers and 9 special sessions in the form of symposia, panels, tutorials and workshops. Authors of the papers and special session participants include members of academia as well as individuals from both the private and public sectors.

Without question, the quality of the meeting depends on the quality of the papers and special session proposals which were submitted, the subsequent review process, and the final selection process. Many individuals committed significant amounts of time reviewing papers and providing constructive feedback to the authors. All of the reviewers did an outstanding job. All submitted papers were competitively double-blind reviewed by at least two reviewers.

The *Proceedings* has several features designed to aid conference participants locate papers of special interest to them. The papers in the *Proceedings* itself are organized alphabetically within their respective tracks. Two other indices appear in the *Proceedings*. The first index is located in the front of the book and alphabetically lists each paper or special session by title within each track. The second index is an author index located in the back of the book.

We extend special thanks to each of you who have volunteered your time in organizing and preparing this year's meeting. We understand the multitude of demands placed on your time and appreciate the personal commitment that each of you has made to the Northeast Decision Sciences Institute.

We encourage you to attend as many sessions as possible. We hope this year's meetings will provide you with much information and that it will serve as a forum for meeting colleagues with similar interests in research and teaching.

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ACCEPTANCE CHARTS FOR LOW DATA ENVIRONMENT

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ABSTRACT

The purpose of acceptance charts is to evaluate a process to determine whether or not it can be expected to satisfy product specifications. Conventional application of acceptance charts involves use of subgroup averages ($\overline{X}$'s), which requires frequent and large amounts of data. This paper, instead, proposes the use of moving averages in the acceptance chart. This way the acceptance chart approach can be used in low data environment. An example is included to illustrate the proposed concept.

INTRODUCTION

There are some processes, which due to their nature, are expected to have unavoidable shifts in their average value but are still able to satisfy customer-established specifications. What is desired in this case is protection against a situation where the process may shift so far from the target value that it will produce some non-conforming products. If the capability standard deviation (defined later in the discussion) is considerably smaller than one sixth of the tolerance width, then the use of acceptance charts should be considered. The question that is being tested under acceptance charts is "Is the process producing acceptable parts?"- not "Is the process in statistical control?" There are several sources of reference material on the subject of acceptance charts, e.g., Freund [2], Duncan [1], and Montgomery [5]. In 1957 Freund [2] published a description of acceptance charts, which use subgroup averages. Also see Holmes and Mergen [3], [4] for different applications of acceptance charts.

The construction of acceptance charts requires that you decide what type of risk you would like to consider when setting up the limits on the chart. There are three ways of doing this:

1. Designs based on a Rejectable Quality Level (RQL) and beta risk;
2. Designs based on an Acceptable Quality Level (AQL) and alpha risk; and
3. Designs based on both an RQL, AQL, beta and alpha.

PROPOSED APPROACH

In processes where the data is limited or accumulates slowly, forming subgroups to calculate $\overline{X}$'s may not be practical. In such processes the time to form a subgroup usually is so long that many things could happen in the process by the time a subgroup is formed. Thus, from a process control point of view, it does not make sense to use $\overline{X}$'s and control charts based on $\overline{X}$'s. Examples of alternative control charts are X and Moving Range charts, Cumulative Sum Charts, EWMA charts, etc.

If the process capability standard deviation is smaller than the 1/6 of the tolerance and if the important issue is whether the process is producing acceptable parts or not, then acceptance chart would be a good alternative to use. Thus, to be able to use this tool in low data environment, we propose that $\overline{X}$'s be replaced with the moving averages (MA).

The capability standard deviation is one, which is independent of changes in average values. It can be estimated, for example, by $\overline{R}/d_2$ or $\bar{s}/c_4$, where $\overline{R}$ is the average of the sample ranges, $\bar{s}$ is the average of the sample standard deviations, and d_2 and c_4 are the correction factors for a given sample size. When moving averages be used, $\overline{R}$ is replaced by average moving range ($\overline{MR}$) and the sample size is usually two.

The moving average process for a sequence of uncorrelated random variables X_i's with a constant mean and variance σ_x^2, is defined in this article as

$$MA_{(n,w)} = \frac{(X_n + X_{n-1} + X_{n-2} + \ldots + X_{n-w+1})}{w} = \frac{\sum_{i=n-w+1}^{n} X_i}{w} \qquad (1)$$

where w is the averaging window and n designates the most recent data point.

We will discuss the AQL type of acceptance chart in this paper and use the upper specification limit (USL) case to demonstrate the principle. The lower specification limit (LSL) case is done in essentially the same fashion. First, set a value known as the Maximum Acceptable Process Mean (MAPM) at $k_1\sigma_x$ below the upper specification limit, where σ_x is the capability standard deviation of the X's. If the X values are roughly Normally distributed, then using $k_1 = 2$, for example, will produce an AQL value of approximately 2.5% (Figure 1).

$$\text{MAPM} = \text{USL} - k_1\sigma_x \qquad (2)$$

Figure 1. Graphical Display of MAPM

Then, add $\frac{k_2\sigma_x}{\sqrt{w}}$ to MAPM to arrive at the Upper Acceptance Limit (UAL) for the sample MA's. The value of k_2 sets the probability of acceptance of a product that has a quality level of AQL. Choosing $k_2 = 2$, for example, would set the probability of accepting a product that has a quality level of AQL to approximately 0.975 (Figure 2).

$$\text{UAL} = \text{MAPM} + \frac{k_2\sigma_x}{\sqrt{w}} \qquad (3)$$

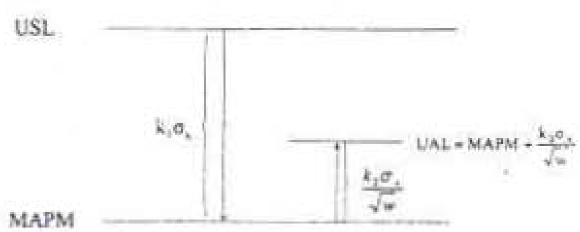

Figure 2. Graphical Display of UAL

Similarly, for the lower specification limit (LSL) the corresponding limits would be determined in the same fashion as

$$\text{LAPM} = \text{LSL} + k_1\sigma_x \qquad (4)$$

$$\text{LAL} = \text{LAPM} - \frac{k_2\sigma_x}{\sqrt{w}} \qquad (5)$$

where LAPM is the lowest acceptable process mean and LAL is the lower acceptance limit. Other values of k_1 and k_2 could be chosen, depending on the desired quality level and the corresponding probability of acceptance. If the MA values fall between UAL and LAL (for two sided specification limits), then the process will be accepted as producing parts which would satisfy the specifications.

EXAMPLE

Let's assume that we have the following one-sided specification limit where the upper specification limit (USL) is equal to 40. Let's also assume that the process output follows a Normal distribution and the capability standard deviation of this process is determined to be 1.0. We will use an averaging window (w) of 2. Below is a demonstration of the calculation of the acceptance limit where we assumed the values of 3 for the constants k_1 and k_2.

$$\text{MAPM} = 40 - 3(1) = 37$$

$$\text{UAL} = 37 + \frac{3(1)}{\sqrt{2}} = 39.12$$

In the above calculations $k_1 = 3$ and $k_2 = 3$ correspond to a 0.9985 probability (roughly) of accepting a 0.15% defective process. Thus MA values less than 39.12 will lead to acceptance of this process, producing items meeting the upper specification limit of 40.

CONCLUSION

In this paper we proposed an acceptance chart using moving averages for a low data environment. This may be used where it is important to make a decision as each data point is accumulated, rather than waiting to form a subgroup.

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